INFERENCE FROM CLUSTERED SAMPLE

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Social scientists often analyze a probability sample of a population rather than a complete census. They then face the task of inferring from sample results to the results they would have obtained if the analysis were performed over the entire population.

One of the keys to making these needed inferences is the availability of methods for estimating the sampling errors of the sample derived population estimates, coupled with generalizations that can be made about the distribution of these population estimates and their estimated sampling errors.

In this paper, we lay out three general methods that may be used to estimate sampling errors of population estimates from complex (clustered and stratified) probability samples. For each of these methods, we describe their implementability and discuss their reliability and validity. This discussion of reliability and validity is based on recently completed empirical research.

For the sake of expositional simplicity, let us assume a sample design that calls for the selection of two primary sampling units (psu's) from each of H strata. It is assumed that there are A primary units in each stratum, and that the selection of two of these A units is made by simple random sampling without replacement. Thus, we have a clustered and stratified sampling design where each population element has equal probability (f) of appearing in the sample (self-weighted sample). It should be noted that any of the three variance estimation methods can be generalized to accomodate unequal allocation between strata, PPS (or any non-epsem) selection of psu's within strata, as well as subsampling of psu's.

To avoid confusion, I will be using the term first-order to describe sample estimates g(S), where g is a function and S is a sample, and corresponding population parameters g(P), where P is the population, that are of primary interest to the substantive analyst. Some examples of these first-order estimates and parameters are ratio means, differences of ratio means, totals, ratios of ratios, simple correlations, partial correlations, multiple correlations, and regression coefficients (simple, multiple, path, MCA, dummy variable, etc.). The term second-order is used to describe estimates, also made from the sample, of the sampling variability (error) of the first-order estimates.

The three second-order estimation techniques described in this paper are labelled the Taylor expansion method (TAYLOR), the method of balanced repeated replication (BRR), and the method of jack-knife repeated replication (JRR). It should be noted, however, that this scheme of appellation is not unique.

[1] *This research was carried out under a Joint Research Project with the U.S. Bureau of the Census. This article draws from results to be found in* (3).

Taylor Expansion Method

The use of the Taylor expansion for obtaining an estimate of the variance of the first-order estimate of a ratio mean has been known for some time. All sampling textbooks describe its use in this context. Deming [2] and Kish [7] describe its use in the propagation of variance for other functions of the basic sample sums. The method is also known as the "delta" or δ-method, or simply as the linearization method. However, to my knowledge, the first detailed published extension, specific to survey sampling, of this method to more complex first-order estimates is due to Tepping [17].

When this method is used, we produce an approximate estimate of the sampling variance of a sample function that is the linear or first term of the Taylor series expansion of the first-order sample estimate of interest.

There are actually two, and sometimes three, approximation assumptions that are made when this method is used. Following Tepping's paper, the method can be described as follows:

Let $y = (y_1,\dots,y_k)$ be a vector of sample statistics that are linear combinations of the primary sampling unit values y_{iha}, where h is the index over strata and a is the psu within strata index. That is,

$$y_i = \sum_{h=1}^{H} \sum_{a=1}^{2} y_{iha} = \sum_{h=1}^{H} y_{ih} . \quad (1)$$

Similarly, let $E(y) = Y = (Y_1,\dots,Y_k)$ be the corresponding vector of population values. Also, let g(Y) be the first-order parameter we wish to estimate by the first-order sample estimation function g(y).

The first assumption to be made is that the sampling variance of g(y) is approximately equal to the sampling variance of the first degree terms of the Taylor approximation of g(y) near Y. That is,

$$\mathrm{VAR}(g(y)) \doteq \mathrm{VAR}(g(Y) + \sum_{i=1}^{k} (y_i - Y_i) \frac{\partial g(Y)}{dY_i}), \quad (2)$$

where the partial derivatives are evaluated at $y = Y$. Since the terms g(Y) and $Y_i(\partial g(Y)/dY_i)$ are constant over all samples, this reduces to

$$\mathrm{VAR}(g(y)) \doteq \mathrm{VAR}(\sum_{i=1}^{k} (\frac{\partial g(Y)}{dY_i}) y_i) . \quad (3)$$

The terms y_i and Y_i are linear combinations of corresponding values of the psu's, thus of the stratum values $y_{ih.}$ and $Y_{ih.}$. Because selection is assumed independent between strata, we may rewrite (3) as

$$VAR(g(y)) \doteq \sum_{h=1}^{H} W_h^2 \, VAR\left(\sum_{i=1}^{k} \frac{\partial g(Y)}{dY_i} y_{ih.}\right) \quad , (4)$$

where W_h is the constant "weight" of the h^{th} stratum.

If there are two primary units selected without replacement and with equal probabilities f_h, from each stratum, then we may estimate the variance of the $y_{ih.}$'s by

$$var\left(\sum_{i=1}^{k} \frac{\partial g(Y)}{dY_i} y_{ih.}\right) =$$

$$(1-f_h)\left\{\sum_{i=1}^{k} \frac{\partial g(Y)}{dY_i} y_{ih1} - \sum_{i=1}^{k} \frac{\partial g(Y)}{dY_i} y_{ih2}\right\}^2 , \quad (5)$$

where y_{ih1} and y_{ih2} are the sample totals from the two psu's of the h^{th} stratum. Keyfitz [6] called early attention to this simple form for two primary sampling units. If $W_h = 1$ and $f_h = f$ for all $h=1,\ldots,H$, our estimate of $VAR(g(y))$ is

$$var(g(y)) =$$

$$(1-f) \sum_{h=1}^{H} \left\{\sum_{i=1}^{k} \frac{\partial g(Y)}{dY_i} y_{ih1} - \sum_{i=1}^{k} \frac{\partial g(Y)}{dY_i} y_{ih2}\right\}^2 . (6)$$

In order to use this estimate, we should ideally have values for the constants $\partial g(Y)/dY_i$. Of course, if these were known, we would probably know $g(Y)$ and would not need to make the estimate $g(y)$. These constants must be estimated from the sample at hand. Thus, we have the second approximation assumption associated with this method.

It is commonly assumed that this substitution of sample values for population values does not greatly increase the error in this estimate of variance. However, this is only a conjecture.

Balanced Repeated Replication (BRR) Methods

More complete descriptions and discussions of balanced repeated replication methods for computing estimates of sampling errors have already appeared in a number of developmental papers [4, 5, 8, 9, 10, 11, 12, 14]. However, as is the case with the other two variance estimation methods, with the exception of an unpublished study by Tepping [16] concerning the behavior of the Taylor expansion estimates of the variance of simple ratios and the research herein described [3], no empirical data have been collected that deal with the validity and precision of these methods.

The variance estimates produced by BRR can be described as follows: Assume that we have a stratified sample design with two primary sampling units selected from each stratum with equal probability f (srs). Let S denote the entire sample; let H_i denote the i^{th} half-sample formed by including one of the two primary units in each of the strata; and let C_i denote the i^{th} complement half-sample, formed by the primary units in S not in H_i.

If we form k half-samples $H_1,\ldots,H_k$ and corresponding complement half-samples $C_1,\ldots,C_k$, then we may produce four BRR type estimates of variance of the first-order estimate $g(S)$.

Half Minus Total - $var_{BRR-H}(g(S)) =$

$$\frac{(1-f)}{k} \sum_{i=1}^{k} (g(H_i)-g(S))^2 \quad (7)$$

Complement Minus Total - $var_{BRR-C}(g(S)) =$

$$\frac{(1-f)}{k} \sum_{i=1}^{k} (g(C_i)-g(S))^2 \quad (8)$$

Sum of BRR-H and BRR-C - $var_{BRR-S}(g(S)) =$

$$\frac{var_{BRR-H}(g(S)) + var_{BRR-C}(g(S))}{2} \quad (9)$$

Half Minus Complement - $var_{BRR-D}(g(S)) =$

$$\frac{(1-f)}{4k} \sum_{i=1}^{k} (g(H_i) - g(C_i))^2 \quad (10)$$

There are several methods for choosing the pattern of primary units that form the repeated half and complement half samples H_i and C_i. The method used in my empirical research is known as "full-orthogonal balance." For a more complete description of the method, see [10, 11].

As previously noted, each of the second-order estimates actually estimates the variance of a linearized form of the first-order estimate. In the case of the four BRR estimates, this linearization of $g(S)$ is

$$\frac{1}{2k} \sum_{i=1}^{k} (g(H_i) + g(C_i)) \quad . \quad (11)$$

Because of the interchangeability of H_i with C_i, the forms BRR-H and BRR-C possess the same expectation. As a result, their mean, BBR-S, shares this equality. The BRR-H and BRR-C forms should be viewed as estimates of BRR-S, which are less costly to compute. For the moment, we will eliminate the -H and -C forms of BRR from our discussion. If the function (11) has bias which is linearly decreasing in the number of primary sampling units, the BRR-S form (as well as the -H and -C forms) gives an unbiased estimate of the mean square error of (11).

Under any circumstances, the form BBR-D is an unbiased estimate of variance for (11) [8, 9].

Jack-Knife Repeated Replication (JRR) Methods

The term jack-knife repeated replication describes a set of second-order estimation methods that were motivated by the Tukey jack-knife estimation procedure [1, 18] and by BRR.

With BRR methods, each of the k replications estimates the variance of the entire sample. With the JRR methods, each replication gives us a measure of the variance contributed by a single stratum. The technique used to measure this stratum variance contribution was suggested by the Tukey jack-knife method for variance estimates formed by leaving out replicates of the sample. The specific procedures described below appear in the literature for the first time here.

JRR estimates of the variance are computed as follows: Assume that we have an epsem, stratified sample design with two primary sampling units selected with equal probability f, from each of H strata. Let S denote the entire sample; let J_i, (i=1,...,H), denote the replicate formed by removing from S one of psu's in the i^{th} stratum, and including twice the other psu in the i^{th} stratum. Let CJ_i, (i=1,...,H) denote the complement replicate formed from S by interchanging the psu's in the i^{th} stratum that are eliminated and duplicated. The four JRR estimates of the variance of the first-order estimate g(S) are:

Estimate I (JRR-H)

$$var_{JRR-H}(g(S)) = (1-f) \sum_{i=1}^{H} (g(J_i)-g(S))^2 \quad (12)$$

Estimate II (JRR-C)

$$var_{JRR-C}(g(S)) = (1-f) \sum_{i=1}^{H} (g(CJ_i)-g(S))^2 \quad (13)$$

Estimate III (JRR-S)

$$var_{JRR-S}(g(S)) = \frac{var_{JRR-H}(g(S))+var_{JRR-C}(g(S))}{2} \quad (14)$$

Estimate IV (JRR-D)

$$var_{JRR-D}(g(S)) = \frac{(1-f)}{4} \sum_{i=1}^{H} (g(J_i)-g(CJ_i))^2 \quad (15)$$

From (15), the linearization associated, in a loose fashion, with the JRR estimates is of the form

$$\frac{1}{2H} \sum_{i}^{H} (g(J_i) + g(CJ_i)) \quad . \quad (16)$$

As is the case with BRR, the JRR forms suffixed with -H and -C share the same expectation with each other and with JRR-S. These two former forms should be considered as cheaper to compute but less precise forms of JRR-S. As we did with BRR-H and BRR-C, the forms JRR-H and JRR-C will be, for the moment, eliminated from our discussion.

Implementability

For all three variance estimation methods (TAYLOR -1 form only; BRR-2 forms, BRR-S and BRR-D; JRR-2 forms, JRR-S and JRR-D), as applied to first-order estimates which are functions of total sample first and second moments, much of the cost of computation is directly related to the number of strata; not to the total sample size. All three methods require only one pass (by the computer) over the entire set of <u>individual cases</u>. In this single pass sums, sums of squares, and sums of cross products, are computed for each of the 2 x H psu's that constitute the entire sample. A simultaneous accumulation over psu's yields intermediate statistics for the total sample. All subsequent computations are performed on these psu and total sample "intermediate statistics."

If the TAYLOR method of variance estimation is used, the intermediate statistics for the total sample, the y_i's in (1), are used to produce the sample estimates of the required partial derivatives $\partial g(Y)/dY_i$. Given these partials, we then use the psu intermediate statistics to form the terms $\sum_{i=1}^{k} (\frac{\partial g(Y)}{dY_i}) y_{iha}$ for each psu. The paired squared differences of these terms (6) yield the estimate of sampling error.

When the BRR method of variance estimation is used, one pass over the set of 2 x H psu intermediate statistics is required to form a half-sample and its complement. The half-sample intermediate statistics are formed by the accumulation of one of the two sets of psu intermediate statistics from each of the H strata. The complement half-sample intermediate statistics are formed by subtracting the half-sample intermediate statistics from the intermediate statistics for the total sample. The required first-order estimates g(S), $g(H_i)$, and $g(C_i)$ are produced from the intermediate statistics from the total sample, the i^{th} half-sample, and the i^{th} complement half-sample. These terms are manipulated as in (9) and (10) to form the BRR-S or BRR-D estimate of sampling error.

The computation required for the JRR estimates of sampling error are essentially the same as those required for BRR, with the exception of the formation of the replicates and complement replicates. To form the i^{th} replicate, we subtract from the total sample intermediate statistics the intermediate statistics from one of the psu's in the i^{th} stratum, and add to this the intermediate statistics from the other psu in the stratum. Reversing the labeling of the psu's within the i^{th} stratum, we repeat this procedure to form the

i^{th} complement replicate.

In terms of time requirements, the TAYLOR method of variance estimation is optimal for relatively simple first-order estimates. This includes simple ratio means, differences of ratio means, and simple ratios of ratios. The TAYLOR method begins to lose its time advantage when the computations required to make sample estimates of the partial derivatives become more time-consuming than the time required to form the half or replicate samples. Although the point at which this occurs is somewhat dependent on the number of strata, we have found that the computation of sampling errors for simple correlation coefficients and simple or multiple regression coefficients is equally time-consuming with all three methods. For even more complex first-order estimates, the expression of the partial derivatives in closed form may be beyond our mathematical ability and in this case we must use either BRR or JRR.

At the University of Michigan Survey Research Center, we have not as yet found these forms for partial and multiple correlation coefficients, although this certainly does not mean that they do not exist.

This final observation points out a strength of JRR and BRR methods for variance estimation. If we can specify the first-order estimate g(S) and if we can assume that g(S) is reasonably close to $(g(H_i) + g(C_i))/2$, then we can compute an estimate of the sampling error of g(S) with BRR or JRR.

Reliability and Validity

So far I have described three methods of estimating sampling errors and have commented on their implementability and relative costs. Now we must deal with the question of how well these estimates perform. It would have been preferable if we had general analytic and non-assymtotic comparisons of these three methods. However, to date, efforts in this area have not yielded useful results. Following a tradition among statisticians that goes back at least as far as 1907, when W.S. Gossett, writing under the name "Student," selected 750 simple random samples from a population of criminals' left middle finger measurements in order to evaluate his theoretical derivation of the distribution of the sample mean divided by its estimated standard error [15], I empirically compared and evaluated all three variance estimation methods, using three clustered and stratified sample designs which called for the paired selection of primary sampling units (approx. 14 elements) from 6 strata (approx. 170 elements), 12 strata (approx. 340 elements) and 30 strata (approx. 847 elements). For a more complete description of this study, which made use of data from the Current Population Survey of the U.S. Bureau of the Census, the reader is directed to Frankel [3]. The three methods (five variants: TAYLOR, BRR-S, BRR-D, JRR-S, JRR-D) were used to estimate the sampling error of simple ratio means, differences of ratios, simple correlations, and multiple regression coefficients. BRR and JRR methods were used to estimate sampling errors for partial and multiple correlation coefficients.

Several criteria were used in evaluating the relative merits of the variance estimation methods. First, we looked at their bias, their variance and their mean squared error. None of the three methods appeared to be singularly optimal under any of these criteria.

Somewhere along the line, we realized that none of these criteria actually told us what we wanted to know. We decided that the designation of a statistically best variance estimation technique should be based on a measure of how well the technique allowed the analyst to make valid inference statements about first-order estimates. Put another way, we dècided that our prime interest was not in variance estimation, per se, but in variance estimation as an imput to inference statements.

For this reason, we chose as our ultimate evaluative criteria the degree to which these three variance estimation techniques would yield estimates, var (g(S)), that made the approximation

$$\frac{g(S) - E(g(S))}{\sqrt{var(g(S))}} \sim t_{(H)} \qquad (17)$$

most valid. For each of the five estimation forms, TAYLOR, BRR-S, BRR-D, JRR-S, and JRR-D, we computed the proportion of times this ratio, $(g(S) - E(g(S))/\sqrt{var(g(S))}$, computed for each sample selected under a particular design, fell within the symmetric limits $\pm$ 2.576, $\pm$ 1.960, $\pm$ 1.645, $\pm$ 1.280, and $\pm$ 1.000. Table 1 shows these proportions when this ratio is distributed exactly as a Student's t random variable, and Tables 2-6 show these proportions for the five different variance estimation forms. Since the expected proportions vary with the degrees of freedom, in this case equal to the number of strata, these proportions are shown separately for each of the three sample designs studied. The proportions were averaged over first-order estimates of the same type. There were 6 means, 12 differences of means (D.MEANS) and simple correlations (CORR.S), 8 multiple regression coefficients (BETAS), 6 partial correlation coefficients (PARTIAL R.S) and 2 multiple correlation coefficients (MULTIPLE R.) involved in these averages.

For all types of first-order estimates studied, we find the average proportions (rounded to two places) produced by the BRR-S estimates (Table 3) agree at least as well, and in most cases better, with proportions predicted by Student's t, than proportions produced by any of the other variance estimation methods (TAYLOR, BRR-D, JRR-S, JRR-D).

Although there is some variability between first-order estimate types and between the various sample designs (sizes), the proportions produced with BRR-S estimates, within symmetric intervals, agree excellently with those predicted by Student's t for all first-order estimates except the multiple correlation coefficients (See Tables 1 and 3).

Although the BRR-S method does produce estimates of variance that are optimal under the criteria we have chosen, we find that the other methods are often very close seconds. A measure of this closeness is given in Table 7 which is derived from Tables 2-6. Happily, this table indicates that when we are dealing with first-order estimates that are ratio means and differences of means, all methods perform about equally well. Thus, given the research at hand, we can tentatively recommend the following optimal (both in terms of computing costs and our chosen statistical criteria) strategy be followed for producing sampling errors of first-order population estimates.

1. Use the Taylor method for ratio means, differences of ratios and other similar forms.
2. Use BRR-S for more complex regression-related statistics; correlations and regression coefficients.
3. Given 1 and 2, one can feel fairly safe in using the approximation

$$\frac{g(S) - E(g(S))}{\sqrt{\mathrm{var}(g(S))}} \sim t_{(H)}$$

in order to generate either classical or Bayesian inference statements.

TABLE 1

PROPORTION OF STUDENT'S T AREA WITHIN SELECTED INTERVALS

Degrees Of Freedom	Intervals				
	± 2.576	± 1.960	± 1.645	± 1.282	± 1.000
6	.9580	.9023	.8489	.7529	.6441
12	.9757	.9264	.8741	.7760	.6630
30	.9848	.9407	.8896	.7903	.6747
∞	.9900	.9500	.9000	.8000	.6827

TABLE 2

SAMPLE ESTIMATE - EXPECTED VALUE, DIVIDED BY TAYLOR ESTIMATE OF STANDARD ERROR
PROPORTION OF TIMES WITHIN STATED LIMITS

Statistic(s)	± 2.576	± 1.960	± 1.645	± 1.282	± 1.000
6 STRATA DESIGN					
Means	0.9483	0.8879	0.8329	0.7379	0.6279
D. Means	0.9450	0.8842	0.8372	0.7381	0.6306
Corr.S	0.9158	0.8367	0.7744	0.6708	0.5631
Betas	0.9421	0.8733	0.8146	0.7167	0.6029
Partial R.S.					
Multiple R.					
12 STRATA DESIGN					
Means	0.9712	0.9192	0.8646	0.7625	0.6542
D. Means	0.9653	0.9078	0.8525	0.7539	0.6358
Corr.S	0.9333	0.8589	0.8028	0.7050	0.5992
Betas	0.9662	0.9121	0.8496	0.7437	0.6217
Partial R.S.					
Multiple R.					
30 STRATA DESIGN					
Means	0.9819	0.9431	0.8881	0.7844	0.6537
D. Means	0.9821	0.9433	0.8842	0.7742	0.6429
Corr.S	0.9650	0.8983	0.8362	0.7225	0.6025
Betas	0.9787	0.9319	0.8837	0.7781	0.6612
Partial R.S.					
Multiple R.					

TABLE 3

SAMPLE ESTIMATE - EXPECTED VALUE, DIVIDED BY BRR-S ESTIMATE OF STANDARD ERROR
PROPORTION OF TIMES WITHIN STATED LIMITS

Statistic(s)	±2.576	±1.960	±1.645	±1.282	±1.000
		6 STRATA DESIGN			
Means	0.9558	0.9042	0.8450	0.7562	0.6450
D. Means	0.9500	0.8997	0.8497	0.7578	0.6483
Corr.S	0.9475	0.8864	0.8358	0.7386	0.6250
Betas	0.9662	0.9150	0.8600	0.7683	0.6642
Partial R.S.	0.9567	0.9083	0.8550	0.7661	0.6511
Multiple R.	0.9350	0.8950	0.8233	0.7383	0.6133
		12 STRATA DESIGN			
Means	0.9721	0.9221	0.8700	0.7692	0.6612
D. Means	0.9658	0.9117	0.8617	0.7614	0.6458
Corr.S	0.9553	0.8967	0.8439	0.7578	0.6397
Betas	0.9733	0.9337	0.8746	0.7733	0.6529
Partial R.S.	0.9661	0.9117	0.8694	0.7544	0.6250
Multiple R.	0.9200	0.8500	0.7900	0.6767	0.5500
		30 STRATA DESIGN			
Means	0.9825	0.9444	0.8906	0.7894	0.6569
D. Means	0.9829	0.9462	0.8875	0.7783	0.6475
Corr.S	0.9725	0.9108	0.8617	0.7533	0.6325
Betas	0.9825	0.9381	0.8900	0.7887	0.6706
Partial R.S.	0.9550	0.8967	0.8442	0.7533	0.6450
Multiple R.	0.9125	0.8250	0.7350	0.6375	0.5275

TABLE 4

SAMPLE ESTIMATE - EXPECTED VALUE, DIVIDED BY BRR-D ESTIMATE OF STANDARD ERROR
PROPORTION OF TIMES WITHIN STATED LIMITS

Statistic(s)	±2.576	±1.960	±1.645	±1.282	±1.000
		6 STRATA DESIGN			
Means	0.9533	0.8996	0.8404	0.7487	0.6379
D. Means	0.9481	0.8950	0.8450	0.7503	0.6436
Corr.S	0.9411	0.8761	0.8189	0.7131	0.6069
Betas	0.9587	0.8996	0.8433	0.7446	0.6446
Partial R.S.	0.9467	0.8900	0.8283	0.7272	0.6111
Multiple R.	0.9033	0.8217	0.7583	0.6417	0.5467
		12 STRATA DESIGN			
Means	0.9721	0.9208	0.8687	0.7667	0.6579
D. Means	0.9656	0.9097	0.8594	0.7586	0.6422
Corr.S	0.9492	0.8883	0.8344	0.7397	0.6264
Betas	0.9700	0.9250	0.8654	0.7646	0.6412
Partial R.S.	0.9583	0.9006	0.8456	0.7267	0.6011
Multiple R.	0.9067	0.8150	0.7400	0.6067	0.5067
		30 STRATA DESIGN			
Means	0.9819	0.9437	0.8894	0.7881	0.6569
D. Means	0.9825	0.9454	0.8867	0.7779	0.6462
Corr.S	0.9696	0.9083	0.8550	0.7467	0.6212
Betas	0.9812	0.9369	0.8881	0.7831	0.6687
Partial R.S.	0.9533	0.8925	0.8350	0.7433	0.6300
Multiple R.	0.8975	0.8100	0.7175	0.6125	0.4975

TABLE 5

SAMPLE ESTIMATE - EXPECTED VALUE, DIVIDED BY JRR-S ESTIMATE OF STANDARD ERROR
PROPORTION OF TIMES WITHIN STATED LIMITS

6 STRATA DESIGN

Statistic(s)	±2.576	±1.960	±1.645	±1.282	±1.000
Means	0.9508	0.8942	0.8362	0.7421	0.6329
D. Means	0.9464	0.8939	0.8397	0.7428	0.6367
Corr.S	0.9311	0.8633	0.8047	0.6992	0.5906
Betas	0.9521	0.8833	0.8304	0.7312	0.6200
Partial R.S.	0.9367	0.8683	0.8100	0.7050	0.5950
Multiple R.	0.9117	0.8400	0.7800	0.6600	0.5600
12 STRATA DESIGN					
Means	0.9712	0.9200	0.8662	0.7650	0.6554
D. Means	0.9653	0.9083	0.8558	0.7561	0.6375
Corr.S	0.9439	0.8750	0.8261	0.7308	0.6167
Betas	0.9675	0.9162	0.8542	0.7496	0.6283
Partial R.S.	0.9494	0.8883	0.8256	0.7106	0.5822
Multiple R.	0.8950	0.8133	0.7383	0.6333	0.5167
30 STRATA DESIGN					
Means	0.9819	0.9431	0.8887	0.7856	0.6537
D. Means	0.9821	0.9433	0.8842	0.7742	0.6433
Corr.S	0.9658	0.9021	0.8471	0.7346	0.6137
Betas	0.9800	0.9325	0.8844	0.7787	0.6631
Partial R.S.	0.9458	0.8792	0.8192	0.7250	0.6183
Multiple R.	0.8950	0.7925	0.7025	0.5950	0.4950

TABLE 6

SAMPLE ESTIMATE - EXPECTED VALUE, DIVIDED BY JRR-D ESTIMATE OF STANDARD ERROR
PROPORTION OF TIMES WITHIN STATED LIMITS

6 STRATA DESIGN

Statistic(s)	±2.576	±1.960	±1.645	±1.282	±1.000
Means	0.9500	0.8912	0.8337	0.7396	0.6329
D. Means	0.9458	0.8889	0.8389	0.7400	0.6353
Corr.S	0.9292	0.8553	0.7944	0.6892	0.5814
Betas	0.9454	0.8796	0.8258	0.7262	0.6142
Partial R.S.	0.9300	0.8561	0.7961	0.6906	0.5772
Multiple R.	0.8850	0.8033	0.7350	0.6133	0.5133
12 STRATA DESIGN					
Means	0.9712	0.9196	0.8658	0.7642	0.6550
D. Means	0.9653	0.9083	0.8544	0.7558	0.6369
Corr.S	0.9428	0.8719	0.8211	0.7217	0.6108
Betas	0.9671	0.9142	0.8508	0.7471	0.6250
Partial R.S	0.9450	0.8800	0.8133	0.6994	0.5694
Multiple R.	0.8850	0.7933	0.7067	0.5933	0.4950
30 STRATA DESIGN					
Means	0.9819	0.9431	0.8881	0.7850	0.6537
D. Means	0.9821	0.9433	0.8842	0.7742	0.6433
Corr.S	0.9658	0.9008	0.8442	0.7333	0.6112
Betas	0.9794	0.9319	0.8844	0.7787	0.6619
Partial R.S	0.9450	0.8775	0.8158	0.7225	0.6108
Multiple R.	0.8875	0.7925	0.6975	0.5825	0.4700

TABLE 7

AVERAGE DEVIATION OF PROPORTIONS FROM THOSE PRODUCED BY BRR-S ESTIMATES
(IN UNITS OF 0.01)

First Order Estimates	Second Order Estimate															
	Taylor				BRR-D				JRR-S				JRR-D			
	Number Of Strata				Number Of Strata				Number Of Strata				Number Of Strata			
	6	12	30	Total	6	12	30	Total	6	12	30	Total	6	12	30	Total
Means	1.2	0.6	0.4	0.7	0.2	0.0	0.0	0.1	1.0	0.0	0.2	0.4	1.2	0.2	0.2	0.5
Differences of Means	0.8	0.8	0.6	0.7	0.0	0.2	0.0	0.1	0.6	0.2	0.6	0.5	1.2	0.2	0.6	0.5
Simple Correlations	5.8	4.0	1.8	3.9	1.8	1.2	0.2	1.1	3.4	2.0	1.4	2.3	4.0	2.8	1.4	2.7
Regression Coefficients	3.2	2.0	0.8	2.0	0.4	0.4	0.4	0.4	1.8	1.4	0.8	1.3	2.0	1.4	0.8	1.4
Partial Correlation Coefficients	-	-	-	-	1.6	1.8	1.0	1.5	3.2	3.4	2.0	2.9	3.4	4.4	2.4	3.4
Multiple Correlation Coefficients	-	-	-	-	7.0	4.0	2.2	4.4	5.4	3.8	3.2	4.1	9.4	6.1	4.4	6.6

NOTE: Total is the average for all three sample designs.

In the few cases where BRR-S proportions were greater than Student's t values, the deviation was measured from the hypothesized t value.

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